X-WINDS: THEORY AND OBSERVATIONS

FRANK H. SHU  
*University of California at Berkeley*

JOAN R. NAJITA  
*Space Telescope Science Institute*

HSIEN SHANG  
*University of California at Berkeley*

and

ZHI-YUN LI  
*University of Virginia at Charlottesville*

We review the theory of x-winds in young stellar objects (YSOs), and we compare its predictions with a variety of astronomical observations. Such flows arise magnetocentrifugally from accretion disks when their inner edges interact with strongly magnetized central stars. X-winds collimate logarithmically slowly into jets, and their interactions with the surrounding molecular cloud cores of YSOs yield bipolar molecular outflows.

I. INTRODUCTION

Strong magnetic fields can considerably enhance the mass loss $M_w$ of thermally driven winds from the surfaces of rapidly rotating stars (Mestel 1968). Even if the resultant flow is quite cold, Hartmann and MacGregor (1982) demonstrated that $M_w$ could have almost arbitrarily large values, dependent only on the ratio of the azimuthal and radial field strengths at the position where the gas is injected onto open field lines near the equator of a protostar that rotates near breakup. Shu et al. (1988) assigned the cause for the protostar to spin at breakup to a circumstellar disk that abuts against the surface of the central object and accretes onto it at a high rate $M_D$. They also replaced Hartmann and MacGregor’s (1982) arbitrary angle of injection for gas velocity and magnetic field direction with the requirement that in steady state, the wind mass loss rate must be a definite fraction $f$ of the disk accretion rate $M_w = f M_D$ (see below).

Blandford and Payne (1982) advanced an influential self-similar model of centrifugally driven winds from the surfaces of magnetized
accretion disks. Pudritz and Norman (1983) applied these pure disk wind models to bipolar outflows, and Königl (1989) investigated how the wind might smoothly join a pattern of accretion flow inside the disk. Heyvaerts and Norman (1989) studied how the winds might collimate asymptotically into jets, while Uchida and Shibata (1985) and Lovelace et al. (1991) advocated alternative driving mechanisms in which magnetic pressure gradients play a bigger role in the acceleration of a disk-blown wind.

Motivated by the problem of binary X-ray sources, a parallel line of research developed concerning how magnetized stars accrete from surrounding disks. Ghosh and Lamb (1978) used order-of-magnitude arguments to show that a strongly magnetized star would truncate the surrounding accretion disk at a larger radius than the stellar radius $R_*$ and divert the equatorial flow along closed field-line funnels toward the polar caps. Although Ghosh and Lamb thought that this inflow would spin the central object up faster than if the accretion disk had extended right up to the stellar surface, Königl (1991) made the surprising and insightful suggestion that the process might torque down the star and account for the relatively slow rate of spin of observed T Tauri stars. Observational support for magnetospheric accretion was subsequently marshaled by Edwards et al. (1994) and Hartmann et al. (1994).

Arons, McKee, and Pudritz (Arons 1986) proposed that a centrifugally driven outflow accompanies the funnel inflow (see also Camenzind 1990). Independently, Basri (unpublished) arrived observationally at the same suggestion by extending the synthesis work by Bertout et al. (1988) on ultraviolet excesses in T Tauri stars. Shu et al. (1994a) put these ideas together into a concrete proposal that generalized the earlier x-wind model of Shu et al. (1988). A related proposal, the so-called “magnetic propeller” (e.g., Li and Wickramasinghe 1997; Lovelace et al. 1999), has been invoked recently to explain the spindown of the cataclysmic variable AE Aquarii (Wynn et al. 1997) and the Rossi X-ray Timing Explorer (RXTE) observations of X-ray pulsars GX 1+4 and GRO J1744-28 (Cui 1997). A quick and somewhat oversimplified summary might be that x-wind theory adds the possibility of outflow to magnetospheric accretion models, while the magnetic propeller idea adds the possibility of time dependence.

II. GENERALIZED X-WIND MODEL

In the generalized x-wind model, if the star has mass $M_*$ and magnetic dipole moment $\mu_*$, the gas disk is truncated at an inner radius,

$$R_x = \Phi_{dx}^{-4/7} \left( \frac{\mu_*^4}{GM_* M_D^2} \right)^{1/7}$$

(1)

A disk of solids may extend inward of $R_x$ to the evaporation radius of calcium-aluminum silicates and oxides (see Shang et al. 1997 and Meyer...
et al. 1997). In equation (1), $\Phi_{\text{dk}}$ is a dimensionless number of order unity that measures the amount of magnetic dipole flux that has been pushed by the disk accretion flow to the inner edge of the disk. In the closed dead-zone model of Najita and Shu (1994) and Ostriker and Shu (1995), $\Phi_{\text{dk}} = 2\bar{B}_w f^{1/2}$, with $f = \dot{M}_w/\dot{M}_D$ and $\bar{B}_w$ defined as the streamline-averaged ratio of magnetic field to wind mass flux. This expression for the coefficient $\Phi_{\text{dk}}$ holds when magnetic flux is swept toward $R_x$ from both larger and smaller radii (see below), and the magnetic flux trapped in a small neighborhood of $R_x$ is $\frac{1}{2}$ times larger than the pure dipole value.

For a Keplerian disk, the inner edge of the disk rotates at angular speed

$$\Omega_x = \left(\frac{GM_*}{R_x^3}\right)^{1/2}$$

(2)

To satisfy mass and angular momentum balance, the disk accretion divides at $R_x$ into a wind fraction, $\dot{M}_w = f \dot{M}_D$, and a funnel-flow fraction, $\dot{M}_* = (1 - f) \dot{M}_D$, where

$$f = \frac{1 - \mathcal{J}_* - \tau}{\mathcal{J}_w - \mathcal{J}_*}$$

(3)

In equation (3) $\mathcal{J}_*$ and $\mathcal{J}_w$ are, respectively, specific angular momenta nondimensionalized in units of $R_x^2 \Omega_x$ and averaged over funnel and wind streamlines, and $\tau$ is the negative of the viscous torque of the disk, $-\mathcal{I}$, acting on its inner edge and measured in units of $\dot{M}_D R_x^2 \Omega_x$. In the approximation that the embedded stellar fields force the x-region to be only weakly differentially rotating (Shu et al. 1994a,b), $\tau$ is small compared to unity.

In steady state, the star is regulated to corotate with the inner disk edge,

$$\Omega_* = \Omega_x$$

(4)

If corotation did not hold, the system would react to reduce the discrepancy between $\Omega_*$ and $\Omega_x$, where $\Omega_x$ is given by equation (2) with $R_x$ determined by how much magnetic field there is to maintain a certain average standoff distance for the stellar magnetosphere [see equation (1)]. For example, suppose the star turns faster than the inner edge of the disk, $\Omega_* > \Omega_x$. The field lines attached to both would then continuously wrap into ever tighter trailing spirals, with the field lines adjacent to the star tugging it backward in the sense of rotation. This tug decreases the star’s angular rate of rotation $\Omega_*$ to more nearly equal the rate $\Omega_x$. Conversely, imagine that $\Omega_* < \Omega_x$. With the star turning more slowly than the inner edge of the disk, the field lines attached to both would continuously wrap into ever tighter leading spirals, with the field lines adjacent to the star tugging it forward in the sense of rotation. This tug increases the star’s
angular rate of rotation $\Omega_\star$, again to more nearly equal the rate $\Omega_X$. In true steady state, $\Omega_\star = \Omega_X$, and the funnel flow field lines acquire just enough of a trailing spiral pattern (but without continuously wrapping up) that the excess of material angular momentum brought toward the star by the inflowing gas is transferred outward by magnetic torques to the footpoints of the magnetic field in the disk (Shu et al. 1994a,b).

The x-wind gas gains angular momentum, and the funnel gas loses angular momentum, at the expense of the matter at the footpoint of the field in, respectively, the outer and inner parts of the x-region. As a consequence, this matter, and the field lines across which it is diffusing, pinch toward the middle of the x-region. In reality, Shu et al. (1997) point out that the idealized steady state of exact corotation probably cannot be maintained because of dissipative effects. Two surfaces of null poloidal field lines (labeled as “helmet streamer” and “reconnection ring” in Fig. 1) mediate the topological behavior of dipolelike field lines of the star, opened field lines of the x-wind, and trapped field lines of the funnel inflow emanating from the x-region. Across each of the null surfaces, which begin or end on “Y-points” (called “kink points” by Ostriker and Shu 1995), the poloidal magnetic field suffers a sharp reversal of direction. By Ampère’s law, large electric currents must flow out of the plane of the figure along the null surfaces. Nonzero electrical resistivity would lead to the dissipation of these currents and to the reconnection of the oppositely directed field lines (see, e.g., Biskamp 1993). The resultant reduction of the trapped
magnetic flux in the x-region as the fan of field lines presses into the evacuated region of the annihilated fields would change the numerical value of the coefficient $\Phi_{\text{ia}}$ in equation (1).

When $R_x$ changes, the angular speed $\Omega_x$ of the footpoint of magnetic field lines in the x-region will vary according to equation (2). However, the considerable inertia of the star prevents its angular velocity $\Omega_\star$ from changing on the timescale of magnetic reconnection at the null surfaces of the magnetosphere. The resulting shear when $\Omega_\star \neq \Omega_x$ will stretch and amplify field lines attached to both the star and the disk. The poloidal field will bulge outward from the increased magnetic pressure, inserting more magnetic flux into the fan of field lines emanating from the x-region [see the simulations of Linker and Mikić (1995) and Hayashi et al. (1996)]. Dynamo action inside the star would presumably replace the upward-rising dipolelike poloidal fields. This dynamo action would be enhanced by the wrapping of field lines between the star and disk. Averaged over long times, we envisage a secular balance, with enough dynamo-generated poloidal field being inserted into the x-region to balance the rate of field dissipation at the null surfaces.

### III. THE CENTRAL MACHINE: CHECKS WITH OBSERVATIONS

In lieu of spatially resolved interferometric images (which will become available within the next two decades), the picture just presented of the central machine that drives collimated outflows from young stellar objects (YSOs) is subject to many different kinds of spectroscopic checks. Here we comment only on four crucial observations: (a) the demonstration that rapidly rotating disks do extend almost to the surfaces of YSOs of sun-like masses, (b) the demonstration that the central stars are magnetized to the extent necessary to truncate the disks at an inner radius $R_x$ compatible with locking stellar rotation rates to their observed low values, (c) the demonstration that the final accretion process onto the central stars is magnetically channeled, and (d) the demonstration that such funnel flows are accompanied by outflows that have the requisite geometry and phase relationship implied by x-wind theory.

#### A. Existence of Rapidly Rotating Inner Disks

The best evidence to date for the existence of inner disks comes from high-resolution spectroscopy of CO overtone emission in young stars (see the chapter by Najita et al., this volume). Through detailed modeling of CO overtone emission it is possible to constrain the outer and, in particular, inner radii for the emission. For example, in the case of the Class I source WL16, the overtone emission is found to arise from disk radii between 5 and 30 $R_\odot$. The outer cutoff of 30 $R_\odot$ probably arises from lack of excitation and has little other physical significance. The inner cutoff of 5 $R_\odot$
may arise either because of CO dissociation or because of a real truncation of the disk at an inner edge.

The latter interpretation is compatible with the emitting area required to explain the strength of the Brackett γ line at line center. The dramatic agreement between the Brγ profile for this source (Najita et al. 1996) and model Brγ spectra for accreting magnetospheres (Muzerolle et al. 1998; see also section III.C below) is suggestive evidence in favor of the claim that WL16 has a strong stellar magnetosphere that truncates its surrounding disk, and through which disk matter accretes onto the stellar surface (Fig. 1). From such evidence, we conclude that one-half of the two crucial ingredients needed for x-wind theory to hold, a rapidly rotating, gaseous, truncated inner disk, does exist in at least one well-studied source found in nature.

### B. Strong Magnetization of the Central Stars

If one eliminates $R_x$ from equations (1), (2), and (4), one gets the magnetic dipole moment of the star,

$$
\mu_* = \Phi_{de} (GM_*)^{5/6} M_D^{1/2} \Omega_*^{-7/6}
$$

required to enforce, in steady state, a given stellar spin rate $\Omega_*$ in a YSO of mass $M_*$ and disk accretion rate $M_D$. We should not expect equation (5) to hold when $M_D$ drops to very low rates, because the remaining reservoir of mass and angular momentum coming through the disk may not then suffice to overcome the inertia of the central star and continue to regulate its spin period. Typical conditions for a classical T Tauri star might be marginal for the purpose at hand: $M_* = 0.5 M_\odot$, $M_D = 3 \times 10^{-8} M_\odot$ yr$^{-1}$, and $\Omega_* = 2 \pi/(8 \ dy)$ (Bertout 1989; Edwards et al. 1993; Gullbring et al. 1998). With $\beta_o \sim 1$ and $f \sim 1/2$, so that $\Phi_{de} \sim 2/\sqrt{3}$, and an equatorial field strength $B_*$ given ideally by $B_* = \mu_* / R_*^3$, where $R_* = 2 R_\odot$ is a typical stellar radius, equation (5) now implies $B_* = 2.1$ kG, a measurable value.

Johns-Krull et al. (1999) have determined the Zeeman broadening of magnetically sensitive lines in the classical T Tauri star BP Tauri. [Several other sources show similarly large fields (C. M. Johns-Krull, personal communication, 1998); see also the early work of Basri et al. 1992.] Johns-Krull and coworkers (1999) have compared their results with the predicted relation (5) and find good agreement. These quantitative measures of the stellar magnetic field buttress the inference from X-ray flaring activity that T Tauri stars are strongly magnetized (Montmerle et al. 1993). One of the more remarkable recent developments in this field is the discovery that even Class I sources are strong X-ray emitters (see the chapter in this volume by Glassgold et al.), indicating that stellar magnetic activity begins early in young stars. We conclude that the other half of the two crucial ingredients needed for x-wind theory to hold, a strong stellar magnetic field, also does exist for several sunlike YSOs found in nature.
C. Magnetically Channeled Accretion onto the Central Star

Since the work of Ghosh and Lamb (1978), workers in the field of X-ray binaries have concurred that if the central object is sufficiently strongly magnetized, the last stages of mass accretion from a surrounding disk may be channeled magnetically onto the central (neutron) star rather than continue to the star’s surface by viscous inspiraling. Only in the case of YSOs, however, do we have direct spectroscopic evidence of this process in the shape of predicted line profiles.

It now appears that the bulk of the hydrogen line emission in young stars arises not in outflowing gas, as was long believed, but in inflowing gas located within several stellar radii of the stellar surface. Outflow signatures are more often seen in Hα and Na I, and inflow signatures are commonly seen in the higher Balmer and Brackett y lines (Edwards et al. 1994; Najita et al. 1996). The comparison of hydrogen line profiles with models of the kinematics and excitation conditions expected in magnetospheric accretion flows supports the idea that the emission originates in hot gas inflowing along stellar magnetic field lines (Muzerolle et al. 1998; see also the chapter in this volume by Najita et al.).

The reader should not conclude from this description, however, that the funnel flow in x-wind theory is identical to the magnetically channeled infall envisioned by Ghosh and Lamb (1978; see also the comments of Shu et al. 1994a and Ostriker and Shu 1995). In Ghosh and Lamb’s (1978) theory, all stellar field lines are closed, and the braking torques exerted by field lines that thread distant radii in the disk balance, in steady state, the accelerating torques exerted by field lines that thread nearby radii in the disk. The “steady state” in this picture is intrinsically statistical and violent in nature (and therefore not computed in any detail), because it involves rapid field wrapping and magnetic reconnection. Because higher latitudes on the star are tugged more slowly by field lines that thread the disk more distantly than field lines from lower latitudes, such a picture would presumably result in a central object that has considerable differential rotation and no well-defined stellar period, contrary to many observations of the rotation properties of classical T Tauri stars (e.g., Vogel and Kuhi 1981; Rydgren and Vrba 1983; Walter et al. 1988; Bouvier et al. 1993; Johns-Krull 1996).

In x-wind theory, closed stellar field lines that thread a nearly uniformly rotating, small, x-region of the disk (funnel-flow field lines) transfer in steady state the excess angular momentum contained in the inflowing matter, not to the star but to the inner portions of the x-region. This excess angular momentum is then removed from the outer portions of the x-region by the opened field lines of the YSO outflow. [Angular-momentum transport from the inner portions to the outer portions of the x-region is probably accomplished by the magnetorotational instability (Balbus and Hawley 1991) and has not yet been examined in any detail.] In pure x-wind theory, the opened field lines came originally from the star (see Fig. 1), and
therefore the magnetocentrifugally driven outflow bears definite geometric and phase relationships to the magnetocentrifugally driven inflow.

D. Geometric and Phase Relationships

As mentioned above, outflow and inflow absorption components are often observed simultaneously and are now believed to arise in the cooler wind and in accretion footpoints, respectively. Periodograms of the hydrogen lines in T Tauri stars can be used to probe the dynamical relationship between the wind, the funnel flow, and the rotation period of the star. Synoptic monitoring by Johns and Basri (1995b) of the T Tauri star SU Aurigae revealed inflow and outflow components to be variable at the photometric (rotation) period of the star but separated in phase by 180°. Johns-Krull and Hatzes (1997) have since found another system with similar properties. If the funnel flow and collimating wind emanate in these objects from the surrounding accretion disks, then the source regions for the flowing gas corotate with the central stars, in agreement with the prediction that \( \Omega_* = \Omega_\chi \). Moreover, the phase lag of 180° is consistent with an origin for both components in the funnel flow and x-wind induced by a tilted magnetic dipole, where field lines in one part of the magnetosphere naturally bend inward, promoting accretion, and field lines 180° away bend outward, promoting mass loss (see Fig. 15 of Johns and Basri 1995b). Unfortunately, the majority of objects studied by Johns and Basri (1995a) do not show such periodicity, indicating that even if \( \Omega_* \) does equal \( \Omega_\chi \) as a long-term average, \( \Omega_* \) may not generally equal \( \Omega_\chi \) instantaneously.

IV. OUTLINE OF MATHEMATICAL FORMULATION

A mathematical formulation of the steady-state problem when \( \Omega_* = \Omega_\chi \) was given in outline by Shu et al. (1988) and in detail by Shu et al. (1994b), who nondimensionalized the governing equations by introducing \( R_\chi, \Omega_\chi, \) and \( \dot{M}_\omega/4\pi R_\chi^2 \Omega_\chi \), respectively, as the units of length, time, and density. Assuming axial symmetry and time independence in a frame that corotates with \( \Omega_\chi \), we may then introduce cylindrical coordinates \((\sigma, \varphi, z)\) and a streamfunction \( \psi(\sigma, z) \) that allows the satisfaction of the equation of continuity, \( \nabla \cdot (\rho u) = 0 \), in the meridional plane:

\[
\rho u_{\sigma\sigma} = \frac{1}{\sigma} \frac{\partial \psi}{\partial \sigma}, \quad \rho u_{\varphi\varphi} = -\frac{1}{\sigma} \frac{\partial \psi}{\partial \sigma} \tag{6}
\]

Field freezing in the corotating frame, \( \mathbf{B} \times \mathbf{u} = 0 \), implies that the magnetic field is proportional to the mass flux, \( \mathbf{B} = \beta \rho \mathbf{u} \), where \( \beta \) is a scalar. The condition of no magnetic monopoles, \( \nabla \cdot \mathbf{B} = 0 \), now requires that \( \beta \) be conserved on streamlines, i.e.,

\[
\beta = \beta(\psi) \tag{7}
\]
Similarly, the conservation of total specific angular momentum requires that the amount carried by matter in the laboratory frame, \( \sigma(\sigma \Omega + u_\phi) \), where \( \Omega \) is replaced by 1 in dimensionless equations, plus the amount carried by Maxwell torques, \(-\sigma B_\psi B\), per unit mass flux, \( \rho u \), equals a function of \( \psi \) alone:

\[
\sigma[(\sigma + u_\phi) - \beta^2 \rho u_\phi] = J(\psi)
\]

Finally, conservation of specific “energy” in the corotating frame for a gas with dimensionless isothermal sound speed \( \epsilon = a_s/R_s \Omega \), results in Bernoulli’s theorem:

\[
\frac{1}{2} |u|^2 + \Upsilon_{\text{eff}} + \epsilon^2 \ln \rho = H(\psi)
\]

Up to an arbitrary constant, defined so that \( \Upsilon_{\text{eff}} = 0 \) at the x-point \( \sigma = 1 \) and \( z = 0 \), \( \Upsilon_{\text{eff}} \) is the dimensionless gravitational potential plus centrifugal potential (measured in units of \( GM_s/R_s = \Omega_z^2 R_s^2 \)):

\[
\Upsilon_{\text{eff}} = \frac{3}{2} - \frac{1}{(\sigma^2 + z^2)^{1/2}} - \frac{\sigma^2}{2}
\]

Equations (7)–(9), with \( \beta, J, \) and \( H \) arbitrary functions of \( \psi \), represent formal integrations of the governing set of equations. The remaining equation for the transfield momentum balance, the so-called Grad-Shafranov equation, cannot be integrated analytically and reads

\[
\nabla \cdot (\mathcal{A} \nabla \psi) = \mathcal{Q}
\]

where \( \mathcal{A} \) is the Alfvén discriminant,

\[
\mathcal{A} = \frac{\beta^2 \rho - 1}{\sigma^2 \rho}
\]

and \( \mathcal{Q} \) is a source function for the internal collimation (or decollimation) of the flow:

\[
\mathcal{Q} = \rho \left[ \frac{u_\phi}{\sigma} f'(\psi) + \rho |u|^2 \beta'(\psi) - H'(\psi) \right]
\]

with primes denoting differentiation with respect to the argument \( \psi \).

The quantity \( \mathcal{A} = 0 \), i.e., \( \beta^2 \rho = 1 \), when the square of the flow speed \( |u|^2 \) equals the square of the Alfvén speed, \( |B|^2/\rho \). For sub-Alfvénic flow, \( \mathcal{A} > 0 \); for super-Alfvénic flow, \( \mathcal{A} < 0 \). If \( \mathcal{A} \) were freely specifiable (which it is not), equation (11) would resemble the time-independent heat conduction equation. What is spreading in the meridional plane of our problem, however, is not heat, but streamlines.
V. FIXING THE FREE FUNCTIONS

When combined with Bernoulli’s equation (9), the Grad-Shafranov equation (11) has three possible critical surfaces associated with it, corresponding to slow magnetohydrodynamic (MHD), Alfvén, and fast MHD crossings (Weber and Davis 1967). Thus, when applied to the problem of the x-wind, equation (11) is a second-order partial differential equation of elliptic type interior to the fast surface and of hyperbolic type exterior to this surface (see Heinemann and Olbert 1978 and Sakurai 1985). The unknown functions $\beta(\psi)$, $J(\psi)$, and $H(\psi)$ are to be determined self-consistently so that the three crossings of the critical surfaces are made smoothly. This does not fix all three functions uniquely, because the loci of the critical surfaces in $(\sigma, z)$-space are not known in advance. It turns out that we can choose one of the loci freely. Alternatively, we can freely specify one of the functions $\beta$, $J$, or $H$. The method of Najita and Shu (1994) fixes in advance the locus of the Alfvén surface and determines all other quantities self-consistently from this parameterization. Shang and Shu (1998) have invented a simplified procedure in which the function $\beta(\psi)$ is specified in advance; then $J(\psi)$, $H(\psi)$, and the loci of the slow, Alfvén, and fast surfaces are found as part of the overall solution (including force balance with the opened field lines of the star and dead zone). The mathematical procedure of choosing an appropriate $\beta(\psi)$ is a substitute for the physical problem of how to load matter onto field lines in the x-region where the ideal MHD approximation of field freezing breaks down (Shu et al. 1994a,b).

Apart from a trivial replacement of $\dot{M}_*\dot{M}_w$, the funnel flow behaves somewhat differently from the x-wind. The funnel flow has a slow MHD crossing but probably no Alfvén or fast MHD crossings. (In steady state the star and disk can communicate with each other along closed field lines by means of the latter two signals.) As a consequence, both $\beta(\psi)$ and $J(\psi)$ can be freely specified for the funnel flow, although $J(\psi)$ must ultimately be made self-consistent with the physical assumption that no spinup or spindown of the star occurs in steady state (that is, that $\Omega_*$ remains equal to $\Omega_\Sigma$ as a long-term average). In these circumstances, when the star is small, Ostriker and Shu (1995) show that $J(\psi)$ is likely to be nearly zero on every funnel-flow streamline $\psi$ = constant. In other words, $\dot{J}_* \approx 0$, and any excess angular momentum brought to the star by the matter inflow is transferred back to the disk by the magnetic torques of a trailing spiral pattern of funnel field lines.

VI. THE COLD LIMIT

The overall problem is mathematically tractable, because the parameter $\epsilon = a_*/R_\Sigma \Omega_*$ is much smaller than unity (typically, $\epsilon = 0.03$). This leads to many simplifications; in particular, to the use of matched asymptotic expansions for solving and connecting different parts of the flow. For ex-
ample, we may show that the slow MHD crossing must be made by matter-carrying streamlines within a fractional distance $\epsilon$ of $R_\star$ (unity in our nondimensionalization), and that to order $\epsilon^2$, $H(\psi)$ may be approximated as zero. In the limit $\epsilon \to 0$, the gas that becomes the x-wind (or the funnel flow) emerges with linearly increasing velocities in a fan of streamlines from the x-region as if from a single point.

In this approximation, the function $\beta(\psi)$ cannot be chosen completely arbitrarily if the magnetic field, mass flux, and mass density do not diverge on the uppermost streamline $\psi \to 1$ as the x-wind leaves the x-region. For modeling purposes, Shang and Shu (1998) adopt the following distribution of magnetic field to mass flux:

$$\beta(\psi) = \beta_0 (1 - \psi)^{-1/3} \quad (14)$$

where $\beta_0$ is a numerical constant related to the mean value of $\beta$ averaged over streamlines:

$$\overline{B} = \int_0^1 \beta(\psi) d\psi = \frac{3}{2} \beta_0 \quad (15)$$

The reader should not worry that equation (14) implies $\beta \to \infty$ as $\psi \to 1$. This singular behavior merely reflects the fact that the magnetic field $B = \beta \rho u$ is nonzero on the last x-wind streamline, where, by definition, $\rho$ must become vanishingly small while $u$ remains finite.

In what follows, we choose $\beta_0$ so that $f \approx \frac{1}{6}$ or $\frac{1}{4}$, with equation (3) implying that $J_w = 3$ or 4 if $J_\star = 0$ and $\tau = 0$. In other words, when the stellar magnetic field is strong enough to truncate the disk via a balanced x-wind and funnel flow, and when one-third to one-fourth of the matter drifting toward the inner edge of the disk becomes entangled in the one-third of the field lines in the x-region that have a proper outward orientation to launch an outflow, then the streamline-averaged location of the Alfvén surface is a factor $\sqrt{J_w} = \sqrt{3}$, or 2 larger than the launch radius $R_\star$. The conclusion that $J_w = 3$ or 4 is important when we compare the predicted terminal velocities of stellar jets, $(2J_w - 3)^{1/2} R_\star \Omega_\star$ averaged over all streamlines, with measured values (see below).

VII. ASYMPTOTIC COLLIMATION INTO JETS

A. Streamline Shape

Given the form $\beta(\psi)$ from equation (14), the loci of streamlines at large distances from the origin may be recovered in spherical polar coordinates $\{r = (\sigma^2 + z^2)^{1/2}, \quad \theta = \arctan(\sigma/z)\}$ from the asymptotic analysis of Shu et al. (1995):

$$r = \frac{2B}{C} \cosh[F(C, 1)] \quad \sin \theta = \text{sech}[F(C, \psi)] \quad (16)$$
where different values of the dimensionless current $C$ correspond to different locations on any streamline $\psi = \text{constant}$, and where $F(C, \psi)$ is the integral function

$$ F(C, \psi) = \frac{1}{C} \int_0^\psi \frac{B(\psi)d\psi}{[2J(\psi) - 3 - 2C\beta(\psi)]^{1/2}} $$

(17)

Notice that $r \to \infty$ when $C \to 0$ and vice versa.

The factor $2\beta$ in equation (16) is replaced by $6\beta$ [see the discussion of Shang and Shu (1998)] if the dead zone is completely opened, as in Fig. 1, rather than completely closed, as assumed in the calculations of Ostriker and Shu (1995) and Shu et al. (1995). In the former case, the hoop stresses of the toroidally wrapped fields on the uppermost streamlines of the x-wind are balanced by the magnetic pressure of a bundle of longitudinal field lines, coming from the star, that carries the same dimensionless magnetic flux, $2\pi\beta$, as their opened counterparts in the x-wind. In the latter case, there are additional longitudinal field lines roughly parallel and antiparallel to the polar axis carrying oppositely directed flux $\pm2\pi\beta$ from the opened field lines of the dead zone. In reality, the system probably alternates between states in which outbursts, similar to coronal mass ejections, completely open the field lines of the dead zone and states in which finite resistivity helps to reconnect the oppositely directed open lines and produce a completely closed configuration for the dead zone.

**B. Density and Velocity Fields**

The density and velocity fields associated with equation (16) are obtained from

$$ \rho = \frac{C}{\beta\sigma^2} \quad v_w = (2J - 3 - 2C\beta)^{1/2} $$

(18)

where $\sigma = r \sin \theta$ and $v_w$ is the wind speed in an inertial frame. The solid and short dashed lines in Fig. 2 show, respectively, isodensity contours and flow streamlines for a case $\beta_0 = 1$ computed on four different scales by the simplified approximate procedure discussed by Shang and Shu (1998). For the model, $J_w = 3.73$, obtainable from the average $\sigma^2$ of the location of the Alfvén surface marked by the inner set of long dashes in Fig. 2 [see equation (8) when $\beta^2\rho = 1$]. With $J_\alpha$ and $\tau$ assumed to be zero, equations (3) and (15) imply $f = 1/7_w = 0.268$ and $\overline{\beta}_w = \frac{3}{2}$. The associated value of $\Phi_{\alpha k} = 2\overline{\beta}_w f^{1/2} = 1.55$.

The outermost density contour in the fourth panel is $\rho = 10^{-8}$ in units of $M_/4\pi\Omega_cR_\odot^3$. The Alfvén and fast surfaces formally asymptote to infinity as the upper streamline $\psi = 1$ is approached, where the density becomes vanishingly small. Because numerical computations are difficult in this limit, the actual uppermost streamline displayed is $\psi = 0.98$ rather
Figure 2. Isodensity contours (solid curves) and streamlines (dotted curves) for a cold x-wind with $\beta(\psi) = \beta_0 (1 - \psi)^{-1/2}$, where $\beta_0 = 1$. Isodensity contours are spaced logarithmically in intervals of $\Delta \log_{10} \rho = 0.5$, and streamlines are spaced so that successive dotted lines contain an additional 10% of the total mass loss in the upper hemisphere of the flow. The loci of the Alfvén and fast surfaces are marked by dashed lines. The empty space inside the uppermost streamline, $\sigma \leq \sigma_1$, is filled with open field lines from the central star that asymptotically have the field strength, $B_c = 2\beta/\sigma_1^2$.

than $\psi = 1$. Although streamlines collimate logarithmically slowly, isodensity contours become cylindrically stratified fairly quickly, roughly as $\rho \propto \sigma^{-2}$ [cf. equation (18)]. Since the radiative emission of forbidden lines is highly biased toward regions of moderately high density, the flow will appear more collimated than it actually is (see Shang et al. 1998).

The dimensional units of length and velocity, $R_\star$ and $\Omega_\star R_\star$, are $\sim 0.06$ AU and $\sim 100$ km s$^{-1}$ in typical application; thus, at a distance of $\sim 600$ AU, about 50% of the streamlines are flowing at speeds $\sim 200$ km s$^{-1}$.
within an angle $\theta \sim 4.5^\circ$ of the polar axis, while another 50% exit in a wide-angle wind. In contrast, high-spatial-resolution imaging with the Hubble Space Telescope shows emission-line jets from young stars appearing to collimate perfectly within tens of AU of the star (e.g., Burrows et al. 1996). If our explanation is correct, the effect is partly an optical illusion arising from the strong cylindrical density stratification of the x-wind flow (see Fig. 2 of Shang et al. 1998).

The relative lack of streamline collimation at tens of AU scales, as opposed to density collimation, provides potentially a key discriminating test of the model. As seen in Fig. 3, the model predicts that forbidden lines arising in the x-wind should have larger velocity widths at the base of the flow, where a typical line of sight encounters flow velocities at a variety of angles, than farther up the length of the jet, where the flow vectors are better oriented along a single direction as the x-wind continues to collimate. Long-slit spectroscopy by Reipurth and Heathcote (1991) and Bacciotti et al. (1996) for several jets shows exactly this behavior. Unfortunately, the light seen near the base of the flow in embedded sources probably arrives by scattering from surrounding dust particles, so the observed effect appears to occur at a larger physical scale than predicted by the theory. Whether the basic kinematics has been affected by observing the phenomenon via “mirrors” remains to be ascertained, perhaps by spectropolarimetry. Better source candidates for a cleaner test of our prediction are the “naked jets” now found in evolved H II regions (B. Reipurth, personal communication, 1998).

The conventional interpretation of line profiles at the base of the flow being wider than in the jet proper invokes a mixing of fast and slow winds (e.g., Kwan and Tademaru 1988; Hirth et al. 1997). Such mixing of fast and slow material might occur because of turbulent entrainment as the lowermost x-wind streamlines interact with a flared accretion disk (Li and Shu 1996a). An x-wind could also interact with a slow wind driven by photoevaporation of a nebular disk (e.g., Shu et al. 1993). In all cases, however, as long as the source of the slower-moving material lies in a flattened distribution, appreciable mixing is easily understood only if the fast flow has a substantial equatorially directed component on the size scale of the inferred accretion disks.

VIII. BIPOLAR MOLECULAR OUTFLOWS

The phenomenon of bipolar molecular outflows (Snell et al. 1980) caught theorists by surprise, because infall rather than outflow had been expected to define the process of star formation. Since 1980, more than 200 molecular outflows have been observed (see, e.g., the reviews of Lada 1985; Fukui et al. 1993; Wu et al. 1996; Cabrit et al. 1997). By now, this ubiquitous phenomenon has become an integral part of the standard paradigm for star formation (Shu et al. 1987). Here, we limit our discussion to recent developments since the publication of Protostars and Planets III.
Figure 3. Position-velocity diagrams for [S II] λ 6731 emission when the synthetic spectrum is taken with a long slit placed along the length of the jet but displaced laterally by 1.5R_0 with respect to its central axis. The different figures correspond to inclination angles i = 90° (top), 60° (middle), and 30° (bottom). The range of projected terminal velocities seen in the models compares well with observed values. By the same token, the fact that measured terminal velocities in stellar jets have a limited range of values implies that the streamlines cannot be launched from a wide variety of disk radii of differing centrifugal speeds, as is implicit in many disk-wind models.
IX. THE AMBIENT MEDIUM INTO WHICH OUTFLOWS PROPAGATE

Low-mass stars form in the dense cores of molecular clouds (Myers 1995). These cores have a typical size \( \sim 0.1 \) pc, a number density \( \sim 3 \times 10^4 \) cm\(^{-3}\), a mass ranging from a small fraction of a solar mass to about 10 \( M_\odot \), and an axial ratio for flattening of typically 2:1. For isolated cores, the last fact implies that agents other than isotropic thermal or turbulent pressures help to support cores against their self-gravity, although it is not yet clear observationally whether the true shapes are oblate, prolate, or triaxial (P. C. Myers, personal communication, 1998). Observed cloud rotation rates are generally too small to account for the observed flattening (Goodman et al. 1993). This leaves magnetic fields, which are believed for other reasons to play a crucial role in contemporary star formation.

In one scenario, the weakening of magnetic support in the central part of a molecular cloud by ambipolar diffusion leads to the continued contraction of a cloud core with ever-growing central concentration (Nakano 1979; Lizano and Shu 1989; Basu and Mouschovias 1994). Li and Shu (1996b) referred to the cloud configuration when the central isothermal concentration first becomes formally infinite as the “pivotal” state. This state separates the nearly quasistatic phase of core evolution from the fully dynamic phase of protostellar accretion (see the first two stages depicted in Fig. 11 of Shu et al. 1987). The numerical simulations indicate that the pivotal states have several simplifying properties, which motivated Li and Shu (1996b) to approximate them as scale-free equilibria with power law radial dependences for the density and magnetic flux function:

\[
\rho(r, \theta) = \frac{a^2}{2\pi G r^2} R(\theta) \quad \Phi(r, \theta) = \frac{4\pi a^2 r}{G^{1/2}} \phi(\theta)
\]  

In equation (19), we have adopted a spherical polar coordinate system \((r, \theta, \varphi)\), and \(a\) is the isothermal sound speed of the cloud, while \(R(\theta)\) and \(\phi(\theta)\) are the dimensionless angular distribution functions for the density and magnetic flux, given by force balance along and across field lines. The resulting differential equations and boundary conditions yield a linear sequence of possible solutions, characterized by a single dimensionless free parameter, \(H_0\), which represents the fractional overdensity supported by the magnetic field above that supported by thermal pressure. Comparison with the typical degree of elongation of observed cores suggests that the overdensity factor \(H_0 = 0.5 - 1\). In Fig. 4, we plot isodensity contours and field lines for the case \(H_0 = 0.5\). The presence of very low-density regions at the poles of the density toroid has profound implications for the shape and kinematics of bipolar molecular outflows. (See also Torrelles et al. 1983, 1994.)
X. MECHANISM FOR MOMENTUM TRANSFER

Observations of one of the earliest types of young stellar objects, the so-called “Class 0” sources, indicate that a high-speed wind is turned on early in the main accretion phase of star formation (Bontemps et al. 1996). Theoretically, the wind should create two wind-blown bubbles, one on each side of the accretion disk, which continues to feed the central stellar object through the equatorial region. The bubbles are bound by the swept-up ambient core material and are expected to become elongated in the direction along the rotation/magnetic axis, where the wind momentum flux is expected to be largest and the ambient density lowest. To determine the shape and kinematics of the swept-up material (i.e., the molecular outflow), one needs to know how momentum is transferred from the wind to the ambient medium. Generally speaking, the ambient medium is swept up by a forward shock that runs ahead of the wind bubble. At the same time, the free wind runs into a reverse shock. Between the shocked ambient medium and the shocked wind material lies a contact discontinuity, across which pressure balance must be maintained.

Depending on the cooling timescale, the shocked material could be either radiative or adiabatic. For an ambient medium shocked with a relatively low shock speed of order 10 km s$^{-1}$ and a relatively high preshock density of order $10^3$ cm$^{-3}$, the cooling timescale is always much shorter than the time to sweep up a mass comparable to the wind mass (Koo and McKee 1992). The short cooling timescale leads to a radiative shocked
ambient medium, which forms a thin shell around the wind bubble. For a wind speed of order 300 km s$^{-1}$ or less, the shocked wind material will become radiative as well (Koo and McKee 1992). With these assumptions, the free protostellar wind is bound by two thin layers of shocked material. The two layers tend to slide relative to each other, exciting Kelvin-Helmholtz instabilities at the interface. Full development of fluid instabilities may lead to a well-mixed shell of two different types of shocked material, creating an essentially ballisitic putty. Each segment of mixed radiative shell absorbs all of the wind momentum imparted to it. This local conservation of vector momentum forms the basis of a simple theory for molecular outflows (Shu et al. 1991; see also Wilkin 1997). A more realistic treatment, where the shell is not treated as spatially thin because of the “cushioning” effect of the embedded magnetic fields, will be more complicated but deserves investigation [see Li and Shu’s (1996a) treatment of the related problem of the interaction of a wide-angle wind with a flared disk].

XI. A SIMPLE THIN-SHELL THEORY

In the simple theory developed by Shu et al. (1991), wind momentum flux per steradian of the form

$$K(r, \theta) = \frac{\dot{M}_w v_w}{4\pi} P(\theta)$$

(20)

propagates into a static molecular cloud core of density given by the first of equations (19). In equation (20), $\dot{M}_w$ and $v_w$ are the mass loss rate and average speed of the wind, and $P(\theta)$ is a normalized angular distribution function. The model makes an implicit assumption that the directions of the spin vector $\Omega_x$ of the accretion disk, which defines the axis of the bipolar outflow, and the ambient magnetic field $B$, which defines the axis of the molecular toroid, are aligned. Presumably this alignment arises because magnetic braking in the prior epoch of molecular cloud core formation was more efficient at reducing components of precollapse $\Omega$ perpendicular to $B$ than at reducing the component parallel to $B$ (see, e.g., Mouschovias and Paleologou 1980).

If we assume the same polar axis for the functions $P(\theta)$ and $R(\theta)$, momentum conservation in every direction $\theta$ yields the expansion velocity of the swept-up shell as

$$v_s = \left(\frac{\dot{M}_w}{2M_a}\right)^{1/2} (av_w)^{1/2} \left[\frac{P(\theta)}{R(\theta)}\right]^{1/2}$$

(21)

where $\dot{M}_a = a^{3/2}G$ is a measure of the mass accretion rate onto the central protostellar object (Shu 1977). Note that the shell expansion speed is independent of radius $r$, a special property of the $r^{-2}$ density profile of
the ambient medium. Aside from factors of order unity, equation (21) predicts a characteristic bipolar molecular outflow speed that is the geometric mean of the wind speed and the cloud sound speed. Since the wind speed $v_w$ is typically a few hundred km s$^{-1}$ (Edwards et al. 1994) and the sound speed $a$ is of the order of a fraction of a km s$^{-1}$, we expect a characteristic molecular outflow speed of order 10 km s$^{-1}$, as observed (Fukui et al. 1993). The simple model also naturally accounts for an otherwise mysterious “Hubble” expansion, i.e., a flow velocity proportional to the distance from the central object that is often observed in molecular outflow lobes (e.g., Lada and Fich 1996).

At the time of their proposal, the functional forms of $P(\theta)$ and $R(\theta)$ were not known to Shu et al. (1991). Masson and Chernin (1992) therefore criticized their model on the grounds that “reasonable” choices for $P(\theta)$ and $R(\theta)$ yielded masses for the swept-up material that should increase with increasing line-of-sight velocities $v$ rather than decline as a power law, $dn/dv \propto v^\alpha$, with the exponent $\alpha$ close to $-1.8$, as determined empirically for several well-observed outflows. Masson and Chernin (1992) argued that such a steep decrease of mass with velocity would be difficult to reproduce with any model of outflow cavities carved by wide-angle winds, in essence because the portion of the lobe that travels fastest also travels farthest and is therefore likely to sweep up the greatest amount of matter, not the least amount. This argument presupposes that the main $\theta$ dependence enters in $P(\theta)$ and not in $R(\theta)$, and that neither angular distribution is extreme.

The assumption that $P$ is a function only of $\theta$ neglects the continuing collimation of the x-wind, which occurs logarithmically slowly according to the asymptotic analysis of section VII. Under the approximation that $C$ can be treated as a constant, rather than a quantity that varies logarithmically with $r$, and that the dependences of $\beta(\psi)$ and $J(\psi)$ on $\psi$ can be ignored for the streamlines ($\psi$ not near 1) that interact most strongly with the ambient cloud, equation (18) implies that $P(\theta) \propto \sin^{-2} \theta$. This extreme behavior for $P(\theta)$, very large near the poles $\theta = 0$ and $\pi$, is paired with an equally extreme behavior for $R(\theta)$, vanishingly small near the poles. For the particular case of $H_0 = 0.5$ shown in Fig. 4, we plot the resulting lobe shape in Fig. 5a and the mass-velocity distribution in Fig. 5b. Both properties are in reasonably good agreement with observations, leading to a refutation of the primary argument that bipolar molecular outflows must be driven by highly collimated jets, despite the objection that such jets have difficulty accounting for the relatively poor collimation of most molecular outflow lobes (Bachiller 1996; Cabrit et al. 1997).

**XII. STABILITY OF PARSEC-LONG JETS**

The small amount of ambient matter swept to high velocities, which solves the original objection of Masson and Chernin (1992) to the bipolar
outflow model of Shu et al. (1991), arises because the heaviest part of the x-wind encounters only the low-density material of the poles of the molecular toroid [where $R(\theta)$ ideally becomes vanishingly small]. As a consequence, it is easy for the x-wind to break out of the molecular cloud core in these directions (see Fig. 5) and, thus, to account for the parsec-long jets discovered by Bally and Devine (1997).

Because the x-wind is launched within a few stellar radii of the star, even logarithmically slow collimation yields highly directed jets at parsec distances ($\sim 3 \times 10^6 R_\star$). For $r = 3 \times 10^6$, equation (16) implies that an undeflected $\psi = 0.50$ streamline is collimated within an angle $\theta = 0.9^\circ$ of the polar axis for the example illustrated in Fig. 3. In other words, at a distance $\sim 1$ pc from the central source, 50% of the undisturbed outflow is encased within a cylinder of radius $\sim 0.015$ pc. At very large distances from the central source the difference between x-winds and pure stellar jets narrows considerably.

It has often been asked how jets can maintain their integrity for such long paths. The glib answer provided by all proponents of magnetocentrifugal driving is that the flow is self-collimated. Self-collimation occurs

![Figure 5](image)

**Figure 5.** (a) Shape of swept-up lobes in the meridional plane for a wind power $\propto \sin^{-2}\theta$ and a core density distribution $\propto R(\theta)$ of a singular model with $H_0 = 0.5$. The dashed line indicates schematically the trajectory of the dense parts of the x-wind seen as optical jets that easily burst through the rarefied polar regions of the surrounding molecular toroid. (b) The logarithm of the swept-up mass at each logarithmic interval of the line-of-sight velocity for inclination angles $\vartheta = 80^\circ$, $45^\circ$ (middle line), and $10^\circ$ with respect to the axis of the bipolar flow. The case $\vartheta = 80^\circ$ has a nearly discontinuous step due to the waist of the two lobes in (a).
by the hoop stresses of the toroidal magnetic fields that grow in strength relative to the poloidal components as the flow distance from the source increases. However, such magnetic confinement schemes have long been known to the fusion community to be unstable with respect to kinking and sausaging (see Fig. 10.6 in Jackson 1975; see also Eichler 1993). A solution to the problem has also long been known and is the principle behind working Tokamaks: the introduction of dynamically important levels of longitudinal magnetic fields along the length of the plasma column to stabilize the kinking and sausaging motions (see Figures 10.7 and 10.8 of Jackson 1975). Such dynamically important levels of longitudinal fields along the central flow axis are exactly what fill the hollow core of the jet in the x-wind model of Fig. 2.

Acknowledgments This research is funded by a grant from the National Science Foundation and by the NASA Astrophysics Theory Program, which supports a joint Center for Star Formation Studies at NASA/Ames Research Center, the University of California at Berkeley, and the University of California at Santa Cruz.

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