I. THE FORMATION OF PROTOPLANETS

It is generally accepted that the planets in our solar system were formed in a flattened gaseous nebula centered around the Sun. In typical star-forming molecular clouds, dense cores are observed to have specific angular momentum greater than $6 \times 10^{20}$ cm$^2$ s$^{-1}$ (Goodman et al. 1993) such that their collapse leads to rotationally supported disks analogous to the primordial solar nebula (Terebey et al. 1984). Between 25 and 75% of the young stellar objects (YSOs) in the Orion Nebula appear to have disks (Prosser et al. 1994; McCaughrean and Stauffer 1994), with typical mass $M_D \sim 10^{-2+1} M_\odot$, temperature $\sim 10^{2+1}$ K, and size $\sim 40 \pm 20$ AU
Recent observational breakthroughs have led to the discovery of Jupiter-mass ($M_{\text{Jup}}$) planets around at least a few percent of nearby solar-type stars (see chapter by Marcy et al., this volume). With the present data, we can assert that planetary formation is robust.

In conventional planetary formation models, the first stage of protoplanetary formation is the rapid buildup of solid cores through the coagulation of planetesimals (Safronov 1969; Hayashi et al. 1985; Lissauer and Stewart 1993). When the core mass increases above a critical value ($\approx 15 M_\oplus$), quasistatic evolution is no longer possible, and a rapid accretion phase begins (Mizuno 1980; Bodenheimer and Pollack 1986), leading to the formation of gaseous giant planets (Pollack et al. 1996; also see the chapter by Wuchterl et al., this volume).

The details of this model are not yet fully worked out, but if we suppose that the protoplanet can accrete gas as efficiently as possible, it will first take in gas in the neighborhood of its orbit, assumed circular at radius $r_p$, until it fills its Roche radius $R_R = (q/3)^{1/3}r_p$, where $q = M_p/M_*$ is the protoplanet central star mass ratio, while orbiting in an empty annulus. The mass of the protoplanet is then given by $M_p = 3[4M_D(r)/3M_*]^{3/2}M_*$, where $M_D(r) = \pi \Sigma r^2$ is the characteristic disk mass within radius $r$, with $\Sigma$ being the surface density. At 5.2 AU and $\Sigma = 200$ g cm$^{-2}$, this gives a mass $M_p \sim 0.4 M_{\text{Jup}}$.

Further mass growth now depends on whether the disk has a kinematic viscosity $\nu$ capable of producing a mass accretion rate $\dot{M}_p$ onto the protoplanet. If $\nu$ is finite, then all of the mass flow through the outer disk should go to the protoplanet. To model the disk viscosity, we adopt the prescription of Shakura and Sunyaev (1973), in which $\nu = \alpha H^2 \Omega$, where $\alpha$ is a dimensionless constant, $H$ is the disk semithickness, and $\Omega$ is the disk angular velocity. The most likely mechanism for providing an effective viscosity in stellar accretion disks is MHD turbulence (Balbus and Hawley 1991), which produces $\alpha \approx 10^{-2}$ in a fully ionized disk. Note, however, that $\alpha$ may vary throughout the disk and be much smaller in its intermediate parts (see the chapter by Stone et al., this volume).

Observationally inferred values of the disk accretion rate $\dot{M}_D \sim 10^{-8} - 10^{-7} M_\odot \text{yr}^{-1}$ (Hartmann et al. 1998) are model dependent and highly uncertain. Nonetheless, for such a fiducial $\dot{M}_D$, a protoplanet may attain a mass $M_p > 10 M_{\text{Jup}}$ within $10^6$ yr. The mass of extrasolar planets is $\sim 1 M_{\text{Jup}}$ (see chapter by Marcy et al., this volume). Unless these planets are preferentially formed in low-mass disks, their growth needs to be terminated or inhibited such that they are unable to accept all the mass that flows through the disk. In this paper we focus on disk-protoplanet tidal interactions as a mechanism for accomplishing this.
A. Protoplanet-Disk Tidal Interactions

A protoplanet exerts tidal perturbations that, for $M_p \sim 1 \text{ M}_{\text{Jup}}$, may be adequate to induce the formation of a gap in the disk near its orbit and thereby start to limit accretion flow onto it (Lin and Papaloizou 1993). In general, the gravitational potential due to the protoplanet may be Fourier decomposed in the form

$$\psi = \sum_{l} \sum_{m=0}^{\infty} \psi_{l,m} \cos[m(\phi - \omega_{l,m}t)]$$  \hspace{1cm} (1)

where $\phi$ is the azimuthal angle, $l$ and the azimuthal mode number $m$ are integers, and $\omega_{l,m} = \omega + (l - m)\kappa_p/m$ is the pattern speed, with $\omega$ and $\kappa_p$ being the angular and epicyclic frequency of the protoplanet, respectively. If the orbit of the planet has a small eccentricity $e$, then $\psi_{l,m} \propto e^{l-m}$ (Goldreich and Tremaine 1980; Shu 1984). For circular orbits, only $l = m$ need be considered.

Both outgoing and ingoing density waves are excited in the disk at the Lindblad resonances, located at $r = r_L$ and where, for $l = m$, $\Omega = \omega \pm \kappa/m$ (Goldreich and Tremaine 1978). Here $\kappa$ is the epicyclic frequency of the gas and $\Omega$ is the disk angular velocity. In a Keplerian disk, $\kappa = \Omega$. The ingoing (outgoing) waves carry a negative (positive) angular momentum flux measured in their direction of propagation as they move away from the protoplanet into the disk interior (exterior). The waves thus carry a positive, outward-propagating, conserved angular momentum flux or wave action $F_H$. In most cases it is reasonable to assume that the waves are dissipated at some location in the disk, where their angular momentum density is deposited (see Lin and Papaloizou 1993 and references therein). In the limit of a cold two-dimensional disk, Goldreich and Tremaine (1978) found for a particular $m$ that

$$F_H = \left[ \frac{m \pi^2 \Sigma r^2}{3 \Omega^2 (m - 1)} \left( \frac{d \psi_{m,m}}{dr} + \frac{2 \Omega \psi_{m,m}}{r(\Omega - \omega_{m,m})} \right) \right]_{r=r_L}$$  \hspace{1cm} (2)

The back reaction torque exerted on the disk interior (exterior) to the planet is $-F_H$ ($F_H$). Thus, the inner disk loses angular momentum while the outer disk gains it; hence, the tendency is to form a gap. Evaluation of the total torque acting on each side of the disk requires the summation of contributions from all the resonances, which, for circular orbits, amounts to summing over $m$. As $m \to \infty$, the location of the resonance approaches the orbit.

However, for a non-self-gravitating disk and large $m$, the waves are sonic in character and thus can exist only farther than a distance $\Delta \sim 2H/3$ from the orbit, beyond which the relative disk flow is supersonic. This results in a torque cutoff (Goldreich and Tremaine 1980; Artymowicz 1993b; Korycansky and Pollack 1993; Ward 1997) for $m > r_p/H$. The
dynamics in the coorbital region $\Delta \approx 2H/3$ are not wavelike and are considered below. Summing over all resonances, the total angular momentum flux carried by the waves is essentially the same as that given by Papaloizou and Lin (1984), namely

$$\dot{H}_T = 0.23q^2 \Sigma r_p^3 \omega^2 (r_p/H)^3$$

which was obtained by direct calculation of the torques for a model disk in which the protoplanet orbited in a gap of radial width $\sim H$.

Using equation (3) to estimate tidal torques and then considering the competition between viscous torques, which tend to fill a gap, and tidal torques, which tend to empty it, Lin and Papaloizou (1979b, 1980, 1986a, 1993) found the following viscous condition for gap opening: $q > 40 \nu/\Omega r_p^2$. They proposed that gap formation would lead to the limitation of accretion, but they were able to consider only empty gaps. With the introduction of powerful numerical finite-difference techniques and computers, it has recently become possible to study gap formation numerically more fully than previously (Artymowicz et al. 1998; Bryden et al. 1998; Kley 1998; and see section I.B). The first results of this work indicate that there is a transitional regime in viscosity for which a gap exists but some accretion still occurs, which is then essentially switched off for small enough viscosity.

In addition, Lin and Papaloizou (1993) pointed out that in a disk with very small viscosity, to form a gap it is necessary that $H < R_R = (q/3)^{1/3} r_p$. This thermal condition can be viewed in several ways. It means that the protoplanet’s gravity is more important than pressure at a distance $R_R$ from the protoplanet; generation of large enough hydrostatic pressure forces would require gradients that would cause a violation of Rayleigh’s criterion. From Korycansky and Papaloizou (1996), it is apparent that the thermal condition is required so that the flow in the neighborhood of the protoplanet is nonlinear enough that shocks are produced, which can provide the dissipation associated with gap opening (see below). This condition was found for a simple 2D model disk with a barotropic equation of state.

With more complicated physics and the introduction of three-dimensional effects (Lin et al. 1990a,b), it may be possible to alter wave dissipation patterns and, thus, angular momentum deposition. However, because of the difficulty of clearing material near the protoplanet (it is difficult to see how linear waves could be dissipated closer than several vertical scale heights), it is likely that gap formation will occur much less readily if the thermal condition is not satisfied.

### B. Embedded Protoplanets and Gaps

The flow around a partially embedded protoplanet has been simulated in the low-$M_\nu$ limit in a shearing-sheet approximation (Korycansky and Papaloizou 1996; Miyoshi et al. 1999). These simulations provide useful
clues on the flow pattern near the protoplanet. A global illustrative model is shown in Color Plate 16, in which we set $q = 10^{-3}, H/r = 0.07$, and $\alpha = 10^{-3}$. The flow pattern can be divided into three regions:

1. A circumplanetary disk is formed within the Roche radius of the protoplanet, with semithickness $\sim |r - r_p|$. In such a geometrically thick disk, the tidal perturbations of the host star induce a two-arm spiral shock wave with an open pitch angle. Angular momentum is efficiently transferred from the disk to the star’s orbit around the planet (equivalent to the planet’s orbit around the star) so that gas is accreted onto the core of the protoplanet within a few orbital periods $P = 2\pi/\omega$.

2. An extended arc is formed in the coorbital region near $r_p$, with gas streaming in horseshoe orbits around the $L_4$ and $L_5$ points. Because these are local potential maxima, viscous dissipation leads to the depletion of this region (Lin et al. 1987).

3. In those regions of the disk that have $|r - r_p| > R_H$, the protoplanet’s tidal perturbation results in the convergence of streamlines to form pronounced trailing wakes, both inside and outside $r_p$. These high-density ridges have been identified by some as “stream accretion,” through which material flows from the disk to the protoplanet (Artemowicz and Lubow 1996). However, the velocity field viewed in a frame rotating with the companion (Color Plate 16) clearly indicates that gas in the postshock region along the ridge line is moving away from the protoplanet.

Simulations of gap formation need to consider model evolution over many orbital periods and have to deal with a large density contrast between the gap region and other parts of the disk. Special care is needed to minimize the tendency of numerical viscosity to produce spurious diffusion into the gap region and thus significantly affect the results. In Color Plate 17, we illustrate the excitation and propagation of waves and the existence of a clear gap for a planet (with $q = 10^{-3}$) interacting with a disk (with $H/r = 0.04$ and $\alpha = 10^{-3}$).

In Figure 1, we plot the growth timescale $M_p/M_{\text{p}}$ as a function of $\alpha$ and $H/r$ for protoplanets of mass 1 and 10 $M_{\text{Jup}}$. We comment that it is not necessary for accretion to stop entirely for gap formation to affect the final mass of a protoplanet. All one needs is that the mass doubling time should become longer than either the disk lifetime or the time for the interior disk to accrete. In the latter case, the planet would approach the central star before it could increase its mass significantly (Ivanov et al. 1998). From Fig. 1 we see that, for $\alpha \sim 10^{-3}$ and $H/r = 0.04$, the mass doubling time for 1 $M_{\text{Jup}}$ is $10^6$ yr. For 10 $M_{\text{Jup}}$ the same results apply for $H/r = 0.07$. The results in Fig. 1 thus suggest that the tidal truncation process operates during planetary formation and is important in determining the final mass of the planet $M_p$. 
II. PROTOPLANETARY ORBITAL EVOLUTION

A. Eccentricity Evolution

Resonant interaction between the disk and the planet can lead to changes in the orbital eccentricity $e$ (Lin and Papaloizou 1993; Artymowicz 1993a). The way in which Lindblad torques cause a small eccentricity to grow has been reviewed by Lin and Papaloizou (1993). If only the Lindblad torques are considered, the disk matter is able to pump eccentricity tidally in a similar way to a rapidly rotating star (see below). However, corotation resonances in the disk (at which the pattern speed of a Fourier component of the tidal perturbation corotates with the disk) also need to be considered (see section I.A). When these resonances exist, a torque is applied to the disk at the corotation radius $r_C$, where $\Omega = \omega_{l,m}$ at a rate given by

$$T_{l,m}^C = -m\pi^2 \frac{e}{2} \left[ \Sigma \left( \frac{d\Omega}{dr} \right)^{-1} \frac{d\Sigma^{-1}}{dr} \left( \psi_{l,m} \right)^2 \right]_{r=r_C}$$

where $\zeta = \kappa^2/(2\Omega \Sigma)$ is the vortensity. The role of corotation resonances depends strongly on the $\zeta$ distribution. When $\Sigma$ vanishes at some disk edge, or when there is a gap near a low-mass perturber, Goldreich and Tremaine (1980) showed that the action of torques due to corotation resonances damps $e$ more effectively than effects due to Lindblad resonances excite it. However, this result does not apply to the situation where a protoplanet is embedded in a disk with no edges or gap and where tidal interaction is linear.

Assuming $d\ln e/d\ln r \sim 1$ in the unperturbed disk, Artymowicz (1993a) suggested that the eccentricity of low-mass embedded protoplanets would be damped by tidal interaction. However, in the minimum-mass
solar nebula model where $\Sigma \propto r^{-3/2}$, vortensity is independent of $r$, and then all corotation torques vanish (Ward 1993), although the torque associated with coorbital Lindblad resonances may persist. For a planet with a sufficiently large mass to open a wide gap, both corotation and coorbital Lindblad resonances would be severely weakened and $e$ could then increase on a timescale $e^2/(de^2/dt) \sim \alpha^{-1}(M_p/M_d)P_p$ (Lin and Papaloizou 1993) which, for massive planets, can be shorter than the inferred disk evolution timescale.

If the growth of protoplanets is limited by the formation of a gap of width $\Delta_e$, tides may cause $e$ to increase until $e \sim \Delta_e/r_p$ when a potential expansion valid for small $e$ assumed in the above analysis breaks down. In a cold disk, the protoplanet’s angular velocity at peri-aphelion would then be greater/less than $\Omega(r_p \mp \Delta_e)$, resulting in a torque reversal. Such a process could provide a limit to the growth of $e$. However, a protoplanet’s increased radial excursion may also cause the gap to widen, in which case $e$ might increase to yet larger values. Analytic treatments and self-consistent numerical simulations in the large-$e$ limit have not been carried out so far.

The characterization of extrasolar planets’ orbits from forthcoming observations will provide some useful constraints on the origin of eccentricity. The coexistence of several planets with similar $e$ but very different $M_p$ (or similar $M_p$ but very different $e$) is not a natural outcome of the excitation of eccentricity by disk tidal interaction, because this would tend to produce an $(e, M_p)$ relation (Artymowicz 1992).

### B. Orbital Migration

Planetary orbital migration is induced by the difference between the inner and outer disk torques, $\Delta \tau$, which react back on the planet, assumed to remain in a circular orbit, such that

$$\frac{d r_p}{dt} = \frac{2 \Delta \tau}{r_p \Omega M_p}$$

Based on recent work indicating that $\Delta \tau$ is almost always negative, Ward (1997) suggested that embedded planets migrate inward on a timescale $\sim 10^5(M_p/M_\odot)^{-1}$ yr. If this process occurs in protostellar disks, protoplanets would migrate towards the stellar surface at $r = R_*$ once $M_p > 1 M_\odot$, because their growth timescale (Lissauer and Stewart 1993) would then become longer than this migration timescale. Resolution of this rapid migration dilemma may require the complete and nonlinear analysis of the disk response to the protoplanet in the corotation regions which may be quite complex (see Color Plates 16 and 17). However, supposing that migration occurs and can be stopped near the star (see below), a scenario leading to the formation of short-period planets has been proposed (see chapter by Ward and Hahn, this volume).

After gap formation, there may still be some residual accretion depending on the value of $\alpha$ (see section 1.B). Torques are still exerted...
between disk and protoplanet, and the imbalance due to differing properties of the disk on either side of the protoplanet causes orbital migration (Goldreich and Tremaine 1980; Lin and Papaloizou 1986b; Takeuchi et al. 1996). The estimated effects of advection of angular momentum through the accretion flow on this process are usually found to be small. For small $M_p$, the protoplanet behaves like a disk particle and so migrates towards the star. For larger masses, the evolution is slower. Nonetheless, the protoplanet is always expected to reach the star before it has time to double its mass (Ivanov et al. 1998).

The above discussion naturally leads to the suggestion that short-period planets were formed at several AU away from their host stars and subsequently migrated to their present location (Lin et al. 1996). However, in situ formation cannot be completely ruled out (Bodenheimer 1997).

A protoplanet’s migration may be terminated near $R_*$ if (1) the disk does not extend down to $R_*$ or (2) the host star induces angular momentum transfer to the protoplanet’s orbit via tidal effects (see section III). For the first possibility, interaction between the disk and an intense ($>1$ kG) stellar magnetic field has been suggested (Konigl 1991) as the cause for the modest observed rotation period ($P_\ast \sim 8$ days) of classical T Tauri stars (Bouvier et al. 1993; Choi and Herbst 1996). Such a magnetic field strength may be consistent with recent measurements (Guenther 1997). In this scenario, the stellar field is assumed to induce a cavity out to the magnetospheric radius ($r_m \sim$ a few $10^{11}$ cm) where $P_\ast$ is equal to the local Keplerian period of the disk. When a protoplanet migrates interior to the 2:1 resonance of the gas at $r_m$, the protoplanet is decoupled from the disk, and its migration is stalled. The main uncertainties here are in the observed distribution of $P_\ast$ (Stassun et al. 1999).

### III. PLANET-STAR TIDAL INTERACTION

As the planet approaches $R_*$, the tides raised on the star or planet by its companion become strong enough that their dissipation leads to orbital evolution. If the distribution of mass of the perturbed object has the same symmetry as the perturbing potential, no tidal torque results from the interaction. However, in general, dissipative processes (e.g., radiative damping or turbulent viscosity) acting on the tides produce a lag between the response of the star or planet and the perturbing potential, enabling mechanical energy to be lost and angular momentum to be exchanged between the rotation of the perturbed object and the orbital motion. Only if the system is circular, synchronous, and coplanar (i.e., the spin of the binary components and that of the orbit are parallel) does the tidal torque vanish (Hut 1980). However, such an equilibrium state is not necessarily stable (Counselman 1973; Hut 1980, 1981).

The response of a star or planet to a tidal perturbation is the sum of two terms: an equilibrium tide and a dynamical tide. The equilibrium tide
(Darwin 1879) is the shape that the perturbed object would have if it could adjust instantly to the tidal potential; it is obtained by balancing the pressure and gravity forces. The dynamical tide contains the oscillatory response of the star or planet. It takes into account the fact that gravity or $g$ modes can be excited in the convectively stable layers of the star or planet and that resonances between the tidal disturbance and the normal modes of the star or planet can occur (Cowling 1941).

In massive close binaries, which have a convective core and a radiative envelope, the dynamical tide cannot be neglected, because tidal friction is caused predominantly by the radiative damping of the tidally excited modes (Zahn 1975, 1977; Savonije and Papaloizou 1983, 1984, 1997; Papaloizou and Savonije 1985, 1997; Goldreich and Nicholson 1989; Savonije et al. 1995; Kumar et al. 1995). In that case, $g$ modes, which are excited mainly near the convective core boundary, propagate out through the envelope to the atmosphere, carrying energy and angular momentum. They are damped close to the surface, where the radiative diffusion time becomes comparable to the forcing period, thus enabling a net tidal torque to be exerted on the star or planet.

In the case of solar-type binaries, which have a radiative core and a convective envelope, it was thought until recently that tidal friction could be well described by the theory of the equilibrium tide, in which only turbulent dissipation of the equilibrium tide in the convective envelope is taken into account (Zahn 1977). However, recent studies have indicated that this is not the case. Even if radiative damping can be ignored, the torque derived using only the equilibrium tide is 4–6 times larger than that taking into account the dynamical tide for binary periods of several days (Terquem et al. 1998). The reason is that the theory of the equilibrium tide can in principle be applied only when the characteristic timescales of the perturbed object are small compared to the forcing period. In a solar-type star, however, the convective timescale in the interior region of the convective envelope is as large as a month. Furthermore, because of the uncertainty over the magnitude of the turbulent viscosity associated with convection, it is not clear that the torque due to turbulent dissipation acting on the full tide in the convective envelope is more important than that due to radiative damping acting on the $g$ modes that propagate inwards in the radiative core (Terquem et al. 1998b; Goodman and Dickson 1998). Here, we focus on the case of a planet orbiting around a solar-type star.

Tides raised on the star by the planet can be analyzed in the limit that the rotational frequency of the star is small compared to the orbital frequency. Then, as a result of tidal friction, the star spins up, the orbit decays (the planet spirals in), and the orbit’s eccentricity, if any, decreases. To calculate the timescales on which this evolution occurs, we need to quantify the dissipative mechanisms that act on the tides. Recent studies (Claret and Cunha 1997; Goodman and Oh 1997; Terquem et al. 1998b) have found that the turbulent viscosity in the convective envelope that is
required to provide the observed circularization rates of main-sequence solar-type binaries (Mathieu 1994) is at least 50 times greater than that simply estimated from mixing length theory for nonrotating stars. This indicates either (1) that the observations are questionable, (2) that solar-type binaries are not circularized through turbulent viscosity acting on tidal perturbations (see Tassoul 1988 and Kumar and Goodman 1996 for other suggested tidal mechanisms), or (3) that dissipation in the convective envelope of solar-type stars is significantly more efficient than is currently estimated (see Terquem et al. 1998b for a more detailed discussion of the uncertainties involved). Here we assume that circularization of solar-type binaries does occur through the action of turbulent viscosity on the tides, and we then calibrate its magnitude so as to account for the observed timescales. Under these circumstances, when the response of the star is not in resonance with one of its global normal modes, the tides are dissipated more efficiently by turbulent viscosity than by radiative damping. In a resonance, radiative damping dominates and limits the response of the star at its surface. Although the planetary companion may go through a succession of resonances as it spirals in under the action of the tides, for a fixed spectrum of stellar normal modes its migration is controlled essentially by the nonresonant interaction. For a nonrotating star, and with the calibration mentioned above, the orbital decay timescale, spinup timescale of the star, and circularization timescale, in Gyr, are (Terquem et al. 1998b):

\[
t_{\text{orb}}^* (\text{Gyr}) = 2.763 \times 10^{-4} \frac{(M_p/M_\star + 1)^{5/3}}{M_p/M_\star} \left( \frac{P_o}{1 \text{ day}} \right)^{13/3}
\]

\[
t_{\text{sp}}^* (\text{Gyr}) = 1.725 \times 10^{-6} \left( \frac{M_p + M_\star}{M_p} \right)^2 \left( \frac{P_o}{1 \text{ day}} \right)^{3}
\]

\[
t_{\text{circ}}^* (\text{Gyr}) = 4.605 \times 10^{-5} \left( \frac{M_p/M_\star + 1}{M_p/M_\star} \right)^{2/3} \left( \frac{P_o}{1 \text{ day}} \right)^{13/3}
\]

where \(P_o\) is the orbital period. This circularization timescale is valid only if the initial eccentricity is not too large. If only the convective envelope of the star, where tidal dissipation occurs, is spun up during tidal evolution, then the spinup timescale has to be multiplied by \(I_c/I\), where \(I_c\) and \(I\) are the moments of inertia of the convective envelope and the entire star, respectively. For the Sun, \(I_c/I = 0.14\).

Tides are also raised on the planet by the star. In contrast to the giant planets of our solar system, Jupiterlike planets on a close orbit are expected to have an isothermal, and thus radiative (convectively stable) envelope (Guillot et al. 1996; Saumon et al. 1996). For these planets, both turbulent dissipation in the convective core and radiative damping in the envelope act on the tides. So far it is not clear which mechanism is more important.
We express the timescales associated with turbulent dissipation of the tides in terms of the parameter $Q$, the inverse of which is the effective tidal dissipation function (MacDonald 1964). Damping of the tides in the convective core of the planet leads to the synchronization of the planet rotation with the orbital revolution on the following characteristic timescale, in Gyr (Goldreich and Soter 1966):

$$t_{sp}^{n,1} \text{ (Gyr)} \sim 4.4 \times 10^{-13} Q \left( \frac{1 \text{ day}}{P_p} - \frac{1 \text{ day}}{P_o} \right) \left( \frac{M_p}{M_*} \right) \left( \frac{P_o}{1 \text{ day}} \right)^2 \left( \frac{a}{R_p} \right)^3$$ \hspace{1cm} (9)

where $R_p$ is the planet radius, $a$ is the semimajor axis of the orbit, and $P_p$ is the initial value of the planet rotational period (when it first undergoes tidal interaction with the star). Because the rotational angular momentum of the planet is in general small compared to its orbital angular momentum, synchronization of the planet occurs before any significant orbital evolution can take place. Once synchronization is achieved, damping of the tides raised in the planet always leads to the decay of the orbital eccentricity on a characteristic timescale, which is, in Gyr (Goldreich and Soter 1966),

$$t_{\text{circ}}^{n,1} \text{ (Gyr)} \sim 2.8 \times 10^{-14} Q \left( \frac{M_p}{M_*} \right) \left( \frac{P_o}{1 \text{ day}} \right) \left( \frac{a}{R_p} \right)^5$$ \hspace{1cm} (10)

The tides raised on the planet do not lead to the decay of the orbit once the planet is synchronized. We note that $Q$ may depend on the tidal frequency as seen by the planet, and therefore on the rotational frequency of the planet. Since $t_{sp}^{n,1}$ is calculated assuming synchronization, the value of $Q$ in the above equation may be different from that used for calculating $t_{sp}^{n,1}$. Orbital evolution and tidal heating of Jupiter’s satellites leads to an estimate of $Q \sim 10^5$–$10^6$ for this planet (Goldreich and Soter 1966; Lin and Papaloizou 1979a; Gailitis 1982). However, it is not yet understood where this value of $Q$ comes from, because turbulent viscosity arising from convection would produce $Q \approx 5 \times 10^{13}$ (Goldreich and Nicholson 1977; see Stevenson 1983 for an alternative). There is, of course, no reason to assume that the $Q$ values of the extrasolar planets are similar to that of Jupiter. First, some of these planets orbit very close to their parent stars and therefore are much hotter than Jupiter. Also, $Q$ may depend on the magnitude and the frequency of the tidal oscillation, in which case it would be different for Jupiter if this planet were synchronously rotating on a closer orbit. Therefore we can only speculate when applying the above formulae to extrasolar planets.

Radiative damping of the tides in the envelope, as described above for massive binaries, gives rise to the synchronization and circularization timescales $t_{sp}^{n,2}$ and $t_{\text{circ}}^{n,2}$, respectively. These timescales have been evaluated (in particular for 51 Pegasi) by Lubow et al. (1997). However, as they pointed out, the asymptotic analysis they use is valid only if the
initial spin rate of the planet is less than half that of Jupiter. Besides, this analysis neglects the effect of rotation on the tides, which is important for near-synchronous planets.

In the context of extrasolar planets, we first comment on the magnitude of the perturbed velocity induced by the tides at the stellar surface. Terquem et al. (1998a,b) have found that, in the case of 51 Peg, this velocity is too small to be observed. This result is insensitive to the magnitude of the stellar turbulent viscosity and is not affected by the possibility of resonance. It also holds for the other extrasolar planets that have been detected so far.

As indicated above, the rotation of planets on a close orbit is almost certainly synchronised with the orbital revolution. For the star to be synchronised in less than 5 Gyr (i.e., $t_{\text{orb}}^* < 5$ Gyr), a 1-(3)-Jupiter-mass planet would have to be on an orbit with a period less than 1.4 (3) day(s). We note that in the case of τ Bootis, the rotation of the star may then have been synchronised as a result of tidal effects. If only the convective envelope of the star is spun up, these periods have to be multiplied by about a factor of 2. Therefore, the observation of anomalous rapid rotators could give further evidence of the presence of close planets and provide some indication on how the dissipation of tides affects the rotation of the star (Marcy et al. 1997).

The circularisation timescale for the orbit is $t_{\text{circ}}$ such that $1/t_{\text{circ}} = 1/t_{\text{circ}}^* + 1/t_{\text{circ}}^{P,1} + 1/t_{\text{circ}}^{P,2}$. We note that, according to the expression of the timescales we have given above, $t_{\text{orb}}^* \approx 6t_{\text{circ}}^*$ and $t_{\text{circ}}^{P,1} > 10t_{\text{circ}}^{P,2}$, so that circularisation can be achieved without a significant orbital decay taking place (in contrast to the statement by Rasio et al. 1996). So far it is not clear which of the timescales $t_{\text{circ}}^*$, $t_{\text{circ}}^{P,1}$, and $t_{\text{circ}}^{P,2}$ is the shortest. For 51 Peg, $t_{\text{circ}}^{P,1} < t_{\text{circ}}^*$ requires $Q < 2 \times 10^5$ (see also Rasio et al. 1996 and Marcy et al. 1997 for estimates of $t_{\text{circ}}^{P,1}$). We point out that for a Jupiter-mass planet, $t_{\text{circ}}^* < 5$ Gyr for $P_o < 3$ days. Therefore, all the Jupiter-mass planets orbiting with a period less than about 3 days should be on a circular orbit. This cutoff period is a lower limit because $t_{\text{circ}}^*$ is an upper limit of the circularisation timescale.

Planets found on eccentric orbits at smaller periods are indicative that the stellar rotation frequency (assumed zero up to now) is large compared to the orbital frequency. Indeed, in that case, tidal friction in the star pumps up the orbital eccentricity, opposing the effect of tidal friction in the planet. However, the above calculations indicate that the planet may have to be on a very close orbit for the tidal friction in the star to be able to increase the orbital eccentricity significantly. Whether the planet can get to such a close orbit is questionable if the disk in which the planet formed did not extend down to $R_s$.

The timescale on which the orbit decays is $t_{\text{orb}}^*$. The condition $t_{\text{orb}}^* < 5$ Gyr requires $P_o < 2$ days for a Jupiter-mass planet. If a planet has not been able to get to such a close orbit (e.g., because the inner parts of the disk
were truncated), orbital decay may not occur as a result of tidal friction, and the planet may not plunge into its parent star. Note too that a giant planet that spirals in towards the star may lose mass through Roche lobe overflow (Trilling et al. 1998). Apart from reducing the planetary mass, conservation of angular momentum results in an outward torque on the orbit, which slows the inward migration. A low-mass planet in a very small orbit may result in this way.

However, several planets may attain short periods. After the firstborn protoplanet emerges and migrates to a few $R_*$, subsequently formed protoplanets (beginning farther out because the formation timescale increases with $r$) may migrate inward, pushing protoplanetary cores ahead of their inward path, until they become trapped at Lindblad resonances. Such resonant trapping is found for the Galilean satellites (e.g., Goldreich and Peale 1966; Lin and Papaloizou 1979) and it occurs if the migration rate is sufficiently slow. It raises the intriguing possibility that planets temporarily parked close to the star because of absence of disk material could be forced to plunge into the host star.

On the other hand, if the star is a rapid rotator (with period shorter than that of the orbit) and has a weak magnetic field, tidal effects transfer angular momentum from the star to the orbit, with increasing efficiency as the orbit is pushed out, until a tidal barrier is produced such that the migration time and inverse orbital decay time balance. Tentatively, using estimates derived from equation (7), an inward migration time of $10^6$ yr could be balanced by tidal effects acting on a 1-M$_\text{Jup}$ planet only if the period were $\sim 0.3$ days, which increases to $\sim 0.7$ days for a 3-M$_\text{Jup}$ planet and a migration time of $10^7$ yr. If planets can survive at such short periods (more likely for massive planets and slow migration rates) and if eccentricity growth is suppressed by tides within the planet, tidal effects might temporarily cause the halting of migration by transferring angular momentum from the star to the inner orbit and from there to any residual disk material via Lindblad torques and so on to outer planets (cf. the situation for Saturn’s rings). Such processes have not yet been worked out in detail. However, a solar-type star is likely to have a small amount of angular momentum in comparison to the disk/planet system in total, and once the star slows down sufficiently, the inward migration must continue.

As a planet eventually becomes engulfed by the host star, it is disrupted by tidal breakup, heating, and ram pressure stripping (Sandquist et al. 1998). Supporting evidence for such phenomena may be found in the supersolar metal abundance of 51 Pegasi (G2 V), 55 $\rho$ Cancri (G8 V), and $\tau$ Bootis (F7 V) (Butler et al. 1997; Gonzalez 1997, 1998). The convective envelope for each of these stars contains a few $10^{-2}$ M$_\odot$. The mixing of 10–40 M$_\oplus$ of planetary heavy element material within the stellar envelope (Zharkov and Gudkova 1991) would lead to a significant metal enrichment there. Because the depth of the convection zone of main-sequence stars decreases significantly with increasing stellar mass, the routine engulfing
of planets by their hosts might lead to a tendency for hotter planetary hosts to show a general overall metallicity enhancement with respect to cooler ones.

IV. LONG-TERM STABILITY OF PLANETARY SYSTEMS

In a relatively massive disk, several giant planets may be formed with $M_p \sim 1–3 \, M_{\text{Jup}}$ and $a > 1 \, \text{AU}$ (Lin and Ida 1997). Their long-term orbital stability determines the dynamical evolution of the system. After the depletion of the disk gas, mutual gravitational perturbation between the planets may gradually increase their eccentricities until their orbits cross each other on a timescale $\tau_x$. Extrapolations of existing numerical results (e.g., Franklin et al. 1990; Chambers et al. 1996) give $\tau_x \sim 10^{12–18}$ yr for the solar system. However, $\tau_x$ would reduce to $\sim 10^{8–12}$ or $10^{5–8}$ yr if all the gas giants had $M_p = 1$ or $2 \, M_{\text{Jup}}$, respectively. More recent simulations and crossing times for subsets of planets, configured as in the solar system but with a reduced solar mass, are given in Duncan and Lissauer (1998). These authors find that the orbits of the giant planets remain stable under the expected solar mass loss for up to $10^{16}$ yr or more. The more the central mass is reduced, the shorter is $\tau_x$.

Since systems of massive planets formed with similar values of $a$ and small $e$ eventually suffer orbit crossing (Lin and Ida 1997), massive eccentric planets may have acquired their orbital properties as a consequence of orbit crossing (Rasio and Ford 1996; Weidenschilling and Marzari 1996; Lin and Ida 1997; Levison et al. 1998). Once orbit crossing occurs, close encounters eventually take place. When these involve equal-mass planets, they produce a velocity perturbation $V$ with magnitude limited by the surface escape velocity $V_{\text{esc}} = 60(M_p/M_{\text{Jup}})^{1/3}(\rho_p/1 \, \text{g cm}^{-3})^{1/6} \, \text{km s}^{-1}$, where $\rho_p$ is the planet’s internal density (Safronov 1969). This results in $e \leq V_{\text{esc}}/(a\Omega) \sim 2(M_p/M_{\text{Jup}})^{1/3}(a/1 \, \text{AU})^{1/2}$. Thus, observed eccentricities can be accounted for.

Supposing that giant planets form at $\sim 1 \, \text{AU}$, massive planets ($M_p \gtrsim 5 \, M_{\text{Jup}}$) with moderately high $e \sim 0.3$ and moderately small $a \sim 0.5 \, \text{AU}$ can be accounted for by orbit crossing and merging. Eccentric planets with $a > 1 \, \text{AU}$ and $M_p \sim M_{\text{Jup}}$ can be produced by orbit crossing followed by ejection. Merger, resulting in $e \lesssim 0.5$, is favored at small $r$, whereas ejection occurs preferentially at large $r$, because the cross section for direct collisions is independent of $r$ whereas that for close encounters is $\propto V^{-4} \propto (r\Omega)^{-4} \propto r^{-2}$ (Lin and Ida 1997). Lin and Ida (1997) find that the orbital properties of a merged body are consistent with those of the planets in eccentric orbits around 70 Virginis and HD 114762.

Numerical simulations of systems with many planets indicate that although some may be ejected, a residual population of eccentric planets ($e \gtrsim 0.5$) may remain bound to the central star at large distances ($a > 100 \, \text{AU}$). Close encounters also excite the relative inclination up to $\sim e/2$
V. EFFECTS OF SECULAR PERTURBATIONS DUE TO A DISTANT COMPANION

A. Kozai Effect

Some extrasolar planets are found in binary systems. For example, a planet is found around 16 Cyg B that orbits at a distance of 1.7 AU from the central star, which is known to have a binary companion, 16 Cyg A. It has been suggested that the high eccentricity \( e = 0.67 \) of 16 Cyg B is excited by the gyroscopic perturbation due to 16 Cyg A (Holman et al. 1997). A similar effect may be caused by the gravitational perturbations from other planetary bodies. Here we discuss the effect of secular perturbations on a long timescale such that a time average may be performed. The orbiting bodies may then be considered as having their mass distributed continuously around their orbits as in the classical theories of Laplace and Lagrange (see Hagihara 1972 and references therein). Semimajor axes do not change under secular perturbation. However, changes to the eccentricity and inclination may be produced.

If orbits are coplanar, in general modest changes to eccentricities are produced if the perturbing bodies are widely separated. An exception to this might occur if secular resonances sweep through the system because of a changing gravitational potential due to disk dispersal or changing stellar oblateness (e.g., Ward 1981). However, if orbits are allowed to have high inclination, large eccentricity changes can occur to the orbit of an inner planet perturbed by a distant body or binary companion.

To consider this gyroscopic perturbation effect, let us consider the simplest case of the motion of an inner planet with mass \( M_p \) perturbed by an outer companion with mass \( M' \) assumed to be in a circular orbit with semimajor axis \( a' \). The outer companion is assumed to contain sufficient angular momentum that its orbit remains fixed, defining a reference plane to which the inner planet orbit has inclination \( i \). We note \( \omega_0 \), the longitude of the apsidal line measured in the orbital plane from the line of nodes.

If the distance of \( M_p \) from \( M_* \) is \( r \), its speed is \( v \), the angle between its position vector and that of \( M' \) is \( \Theta \), and only the dominant quadrupole
term in the interaction potential multipole expansion is retained, its motion
is governed by the Hamiltonian (Kozai 1962; Hagihara 1972; Innanen et
al. 1997):
\[
H = \frac{1}{2}M_\ast v^2 - \frac{GM_\ast M_p}{r} - \frac{GM' M_p}{2a^3}(3 \cos^2 \Theta - 1) \tag{11}
\]
After performing a time average, appropriate for secular perturbations,
\[
H = -\frac{GM_\ast M_p}{2a} - \frac{GM' M_p a^2}{16a^3}
\left[\frac{3 \cos^2 i - 1}{(2 + 3e^2)}(2 + 3e^2) + 15e^2 \sin^2 i \cos 2\omega_i \right]
\tag{12}
\]
The fact that both \( H \) and the component of angular momentum parallel to
the outer orbital axis, \( L \cos i = \sqrt{GM_\ast a(1 - e^2)} \cos i \), are constant en-
ables elimination of \( i \) and \( \omega_i \). A complete solution can then be found from
the equation for the rate of change of \( e \), which can be derived from the
canonical equation \( dL/dt = -\partial H/\partial \omega_i \) in the form
\[
\frac{de}{dt} = \frac{15nM' a^3 \sin^2 i \sin 2\omega_i}{8M_\ast a^3} \tag{13}
\]
with \( 2\pi/n \) being the orbital period of the planet \( M_p \).

The above equation indicates that \( e \) can oscillate between extremes
occurring when \( \sin 2\omega_i = 0 \). There has been particular interest in finding
conditions under which \( e \) can start from very small values and then
increase unstably to values close to 1. To examine conditions for this to
occur, we use \( H = \text{const} \) and \( L \cos i = \text{const} \) to find \( \omega_i \) and \( i \) in terms
of \( e \). Assuming initial values \( i = i_0 \) and \( e = e_0 \), when \( \omega_i = \omega_0 \), for in-
finitesimally small but nonzero values of \( e_0 \) that can be neglected, we find
\[
\cos 2\omega_0 = \frac{1 - 5 \cos^2 i_0 - e^2}{5(1 - e^2 - \cos^2 i_0)} \tag{14}
\]
From equation (14), we see that when \( e \to 0 \),
\[
\cos 2\omega_0 = \frac{(1 - 5 \cos^2 i_0)}{5(1 - \cos^2 i_0)} \tag{15}
\]
Clearly, for solutions of the type we seek, we require \( \cos 2\omega_0 > -1 \), which
requires that the initial inclination exceed a critical value (see Innanen et
al. 1997) such that
\[
\cos^2 i_0 < \frac{1}{5} \tag{16}
\]
If this inequality is satisfied (if it is not, then \( e_0 \) cannot be neglected), equation (13) indicates that \( e \) grows from a very small value up until the value obtained by setting \( \omega_0 = \pi/2 \), namely

\[
e^2 = 1 - 5 \cos^2 \frac{i_0}{3}
\]  

attained when \( \cos^2 i = \frac{3}{5} \). A range of eccentricities may be generated in this way. However, values close to unity may be attained for initial inclinations close to 90°. There are then solutions that have large-amplitude oscillations in eccentricity (Kozai effect), and adding the nonsecular terms back in can lead to chaotic behavior. The large eccentricity changes occur independently of the size of the perturbation. However, the characteristic timescale for the eccentricity changes to occur is given from equation (13) as \(- (M_* a^3)/(M' a^3)\) orbital periods of the inner planet, which is long for small perturbations.

Note, however, that the effect requires the motion of the apsidal angle \( \omega_0 \) be governed only by the perturbation considered. Other effects may disrupt this, such as general-relativistic corrections (e.g., Holman et al. 1997) and the oblateness of the central star. Both could be considered in the following discussion, but here we limit ourselves to considering the effects of relativistic apsidal precession, which may lead to constraints on the orbital elements of planets with short periods.

B. Relativistic Effects

To incorporate relativistic apsidal precession to lowest order, we modify the Newtonian potential such that Keplerian elliptic orbits are induced to precess at the correct rate. We modify the potential due to the central mass such that

\[
\frac{-GM_*}{r} \rightarrow \frac{-GM_*}{r} \left( 1 + \frac{3GM_*}{rc^2} \right)
\]

with \( c \) being the speed of light. The additional term can be added into the Hamiltonian \( H \) and averaged so that equation (12) becomes

\[
H \rightarrow H - \frac{3(GM_*)^2 M_p}{2a^2 c^2 \sqrt{(1 - e^2)}}
\]

The same procedure as described above can be used to find \( \omega_0 \) and \( i \) in terms of \( e \) and hence conditions that must be satisfied in order that large values of \( e \) might be attained. The condition analogous to (16) for the Kozai effect to work is

\[
\cos^2 i_0 < \frac{3 - 2f}{5}
\]  

(20)
Here the parameter

\[ f = \left( \frac{GM_\star}{a c^2} \right) \left( \frac{M_\star a^3}{M' a'^3} \right) \]  

(21)

measures the importance of relativistic precession relative to the perturbation due to the outer companion. From this, if \( f > \frac{2}{3} \), the effect is completely suppressed. When the Kozai effect occurs, the maximum possible eccentricity that can be generated is obtained by setting \( \omega_0 = \pi/2 \) and \( i_0 = 90^\circ \). This gives

\[ e^2 = \frac{1}{2} + \frac{4f}{3} + \frac{1}{4} - \frac{4f}{3} \]  

(22)

which indicates that, for \( f = 0.1 \), \( e \) as high as 0.99 can be generated for \( i_0 = 90^\circ \). The parameter \( f \) may be conveniently expressed as

\[ f = 5.0519 \times 10^{-7} \left( \frac{P_o'}{P_o} \right)^2 \left( \frac{1 \text{ day}}{P_o} \right)^{2/3} \]  

(23)

Here, \( P_o \) and \( P_o' \) denote the orbital periods of \( M_\star \) and \( M' \), respectively, and \( q' = M'/M_\star \).

Planets with periods less than 3–4 days are likely to have their orbits circularized as a result of tidal effects (see above). From the requirement \( f > \frac{2}{3} \), we see that for inner planets with somewhat larger periods (but still on the order of days), binary companions with mass ratio of order unity need orbital periods shorter than about 40 yr in order to pump significant eccentricity. If a massive planet with \( q' \sim 10^{-2} \) is considered, it must orbit with a period ten times shorter, \( \sim 4 \) yr. Such an object should be readily detectable.

We comment that for a given distant binary companion with inclination close to 90° to the planet orbit, and no other perturbers, there is a planet with a period long enough such that the Kozai effect could produce eccentricities close enough to unity that the planet has arbitrarily close approaches to the central star. Then tidal effects may act towards circularization, so decreasing the distance to apoapsis until a circular orbit is produced. The final period would have to be a few days. Alternatively, should the closest approach to the star be beyond the largest radius at which orbits can be circularized, the orbit would retain a Kozai cycle with high maximum eccentricity.

**VI. SUMMARY**

Here we summarize the various processes discussed in this chapter, their influence on the evolution of planetary systems, and their observational implications.
1. **Planet-disk tidal interaction** excites potentially observable spiral density waves and can create clean gaps in the disk, limits mass growth, and drives orbital migration. Growth limitation by gap formation, together with inward migration, may naturally account for the upper limit in the observed mass ratio distribution of extrasolar planets, provided they were formed in disks with \( H \approx 0.1 r_p \) and \( \alpha \approx 10^{-3} \). Because the critical mass for gap formation is an order of magnitude larger than for dynamical gas accretion, a bimodal mass distribution with a depression between these masses may be anticipated. After gap formation, massive protoplanets are expected to migrate with the viscous evolution of the disk until they encounter a stellar magnetospheric cavity or tidal barrier. The existence of short-period planets around classical T Tau stars would imply that the local disk viscous evolution timescale is shorter than the typical age (~10^6 yr). Orbital migration may lead to stellar consumption of protoplanets with surviving planets locked into commensurable orbits. Contamination of host stars (during the latter’s main-sequence evolution) by plunging planets may have led to a relatively high metallicity. Planet-disk interaction is likely to excite only small \( e \approx 0.3 \) and lead to a relation between \( M_p \) and \( e \) that could be observed. A planet with larger eccentricity would overrun the gap and induce nonlinear dissipation.

2. **Stellar tides raised on short-period planets** should rapidly synchronize their spin with that of the orbit. Short-period planets with \( M_p \sim M_{\text{Jup}} \) and \( P \approx 3-4 \) days (e.g., \( \tau \) Boo) should have their orbits circularized by *tides raised on a slowly rotating star* within a few Gyr. In the case of massive planets, the stellar rotation may also be synchronized with the orbit. Only very short-period planets (\( P \approx 2 \) days), however, are expected to undergo significant orbital decay.

3. **Long-term gravitational interaction between planets** with initially small eccentricity leads to orbit crossing on a timescale that is sensitively determined by their masses and initial separations. Subsequent close encounters between comparable masses can lead to high orbital eccentricity and planets scattered into extended orbits. This mechanism may provide a supply of planets with a range of short periods, some of which may undergo tidal circularization or plunge into the star, together with outlying accomplices. The latter could be imaged by the next generation of IR interferometers.

4. **Secular perturbations by distant binary**, or massive outlying planetary companions in relatively highly inclined orbits can also excite high eccentricity through the Kozai mechanism. Potentially this process might produce a supply of short-period planets (\( P \approx 2-3 \) days) that have undergone tidal circularization. Because of the very high relative inclinations required, this mechanism is likely to work only in a small number of cases.

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